1. (6 points) Information about two functions is provided in the following table:

x	-1	0	1	2	3	4
$f(x)$	4	6	3	1	-1	1
$g(x)$	7	0	4	2	2	3

Compute each given operation below and enter your answer in the corresponding box.

(a) $g(f(-1)) =$ $\boxed{13}$

$= g(4)$

$= 3$

(b) $f(g(1)) =$ $\boxed{1}$

$= f(4)$

$= 1$

(c) $\frac{g(1)}{f(1)} =$ $\boxed{\dfrac{4}{3}}$

2. It costs 3 dollars for the Hilbert hotel to make a breakfast sandwich. So if the hotel charges x dollars for a sandwich, the profit per sandwich is given by $p(x) = x - 3$. On a given day, the number of sandwiches that they will sell for x dollars is $n(x) = 180 - 15x$.

(a) (3 points) Consider the function $g(x) = p(x) \cdot n(x)$, and note that
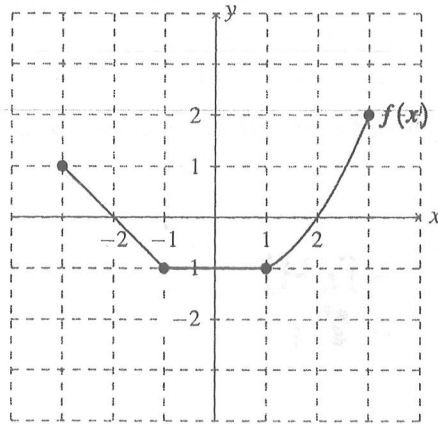
$$g(x) = (x - 3)(180 - 15x).$$

In the context of the problem, interpret the meanings of the inputs and outputs of $g(x)$.

(b) (3 points) Find the roots of $g(x)$ and interpret their meaning in the context of this problem.
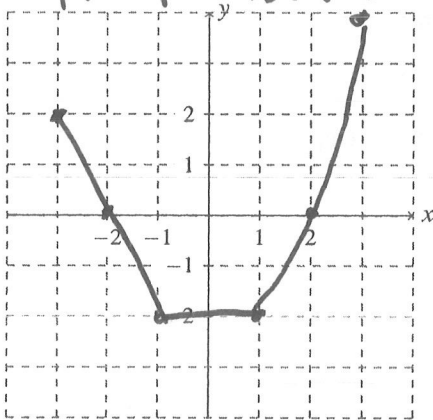
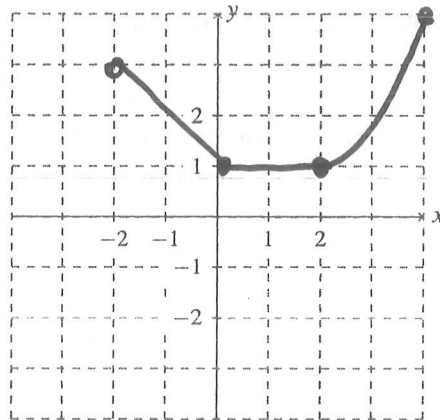3. (12 points) Below is a graph of a function $f(x)$:

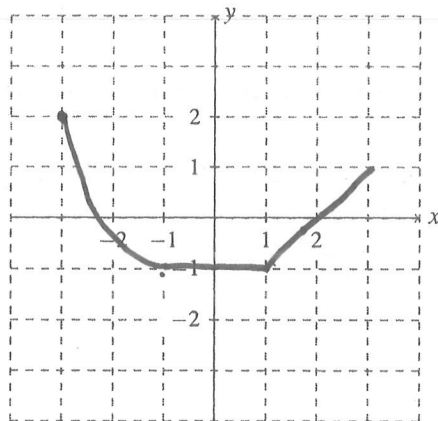On each of the following axes, graph the transformation of $f(x)$ indicated beneath the axes.

Pick points see what happens to them

(a) Draw a graph of $p(x) = 2f(x)$.

(b) Draw a graph of $g(x) = f(x-1)+2$.

(c) Draw a graph of $q(x) = f(-x)$.

4. Suppose that $f(x)$ is a function and consider the transformation

$$g(x) = -3f\left(\frac{1}{2}x - 2\right) + 4.$$

(a) (3 points) State the horizontal transformations in the correct order.

This is in standard form

① Horizontal stretch by 2

② Shift right by 2

(b) (3 points) State the vertical transformations in the correct order.

Can flip these ↗↓

① Reflection across x-axis

② Vertical stretch by 3

③ Shift up by 4.

(c) (2 points) Suppose now that $f(1) = 2$. State one point that must be on the graph of $g(x)$.

The idea is to get $\frac{1}{2}x - 2 = 1$ so that we can say something about $f(\frac{1}{2}x - 2)$

$$\frac{1}{2}x - 2 = 1 \implies x = 6$$

So, if $x = 6$

$$g(6) = -3f\left(\frac{1}{2}(6) - 2\right) + 4 = -3f(1) + 4 = -6 + 4$$
$$= -2$$

So, $(6, -2)$ is a point on $g(x)$.

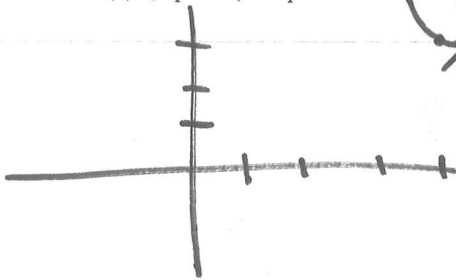5. Answer the following questions about quadratic equations

(a) (2 points) Write $f(x) = x^2 - 7x + 12$ in factored form.

$$f(x) = (x-4)(x-3)$$

(b) (2 points) What is the y-intercept of $f(x) = x^2 - 7x + 12$?

$(0,12)$ MUST BE A POINT!

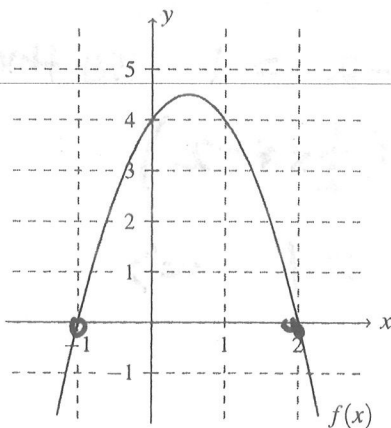(c) (3 points) Explain how we know that $g(x) = 2(x-4)^2 + 3$ has no x-intercepts.

vertex $(4,3)$

$a > 0$

The function faces upwards and starts at $(4,3)$. It's too high!

(d) (3 points) Find a formula for the quadratic function $h(x)$ graphed below. Notes that $h(2) = 0$, $h(-1) = 0$ and $h(0) = 4$. Leave your answer in factored form.

Graph of $h(x)$

Root form!

$$f(x) = a(x+1)(x-2)$$

Use the point $(0,4)$ to find a.

$$f(0) = 4 = a(0+1)(0-2)$$

$$4 = -2a$$

$$a = -2$$

So, $\boxed{f(x) = -2(x+1)(x-2)}$

6. Suppose that an astronaut hits a golf ball while on the moon. The height of the ball (in meters) t seconds later is given by

$$h(t) = -0.8125t^2 + 6.5t .$$

This is a!

(a) (5 points) Put $h(t)$ in vertex form, find the vertex, and interpret the meaning of the vertex in the context of this problem.

coord

x coordt. of vertex : $\dfrac{-b}{2a} = \dfrac{-6.5}{-2 \times 0.8125} = \dfrac{6.5}{1.625} = 4$

y cord. of vertex : $h(4) = -0.8125 \cdot 16 + 26$

$= -13 + 26$

$= 13$

So, vertex is $(4, 13)$ Hence

$h(t) = a(x-4)^2 + 13$ $a = -0.8125$

So, $\boxed{h(t) = -0.8125(x-4)^2 + 13}$

(b) (5 points) Find the t-intercepts of $h(t)$ and interpret the meaning in the context of the problem.

$h(t) = -0.8125t + 6.5t$

$= t(-0.8125t + 6.5)$

$t = 0$ $t = \dfrac{6.5}{0.8125}$

Times when the Ball is on the ground

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7. Suppose that $d(t)$ gives number of doughnuts sold at the Torus (a coffee and doughnut shop) on day t where t is the number of days since they opened.

   (a) (4 points) Let $f(t)$ be the number of doughnuts that the Torus makes on day t. The Torus makes five more than double the number of doughnuts on day t as were sold on the same day one week earlier. Write $f(t)$ as a transformation of $d(t)$ and list the transformations in the appropriate order.

   $$f(t) = 2d(t-7) + 5$$
   ① Shift right 7
   ② Vertical stretch by 2
   ③ Shift up by 5.

   (b) (4 points) Suppose that the profit (in dollars) is given by $p(t)$ on day t. Each day, the Torus makes $1.50 for every doughnut sold, but it costs $200 a day to run the business. Write $p(t)$ as a transformation of $d(t)$ and list the transformations in the appropriate order.